

Primordial inflation from gravity's rainbow

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Abstract

In a recent paper, which has been published in Nature, the LIGO Scientific Collaboration (LSC) obtained an upper limit on the stochastic gravitational-wave background of cosmological origin by using the data from a two-year science run of the Laser Interferometer Gravitational-wave Observatory (LIGO). Such an upper limit rules out some models of early Universe evolution, like the ones with relatively large equation-of-state parameter and the cosmic (super) string models with relatively small string tension arising from some String Theory's models. It results also an upper limit for the relic stochastic background of gravitational waves (RSBGWs) which is proposed by the Pre-Big-Bang Theory.

On the other hand, the upper bound on the RSBGWs which is proposed by the Standard Inflationary Model is well known and often updated by using the Wilkinson Microwave Anisotropy Probe (WMAP).

The potential detection of such a RSBGWs is the only way to learn about the evolution of the very early universe, up to the bounds of the Planck epoch and the initial singularity. This is a kind of information that is inaccessible to standard astrophysical observations.

By using a conformal treatment, a formula that directly connects the average amplitude of the RSBGWs with the Inflaton field has been recently obtained in our paper Gen. Rel. Grav. 42, 5, 1323-1333 (2010). In this proceeding, by joining this formula with the equation for the characteristic amplitude h_C for the RSBGWs, the upper bounds on the RSBGWs from the WMAP and LSC data will be translated in lower bounds on the Inflaton field.

The results show that the value of the Inflaton field that arises from the WMAP bound on the RSBGWs is totally in agreement with the famous *slow roll* condition on Inflation, while the value of the Inflaton field that arises from the LSC bound on the RSBGWs could be not in agreement with such a condition.

1 Introduction

The scientific community aims in a first direct detection of GWs in next years (for the current status of GWs interferometers see [1]) confirming the indirect, Nobel Prize Winner, proof of Hulse and Taylor [2].

Detectors for GWs will be important for a better knowledge of the Universe and either to confirm or to rule out, in an ultimate way, the physical consistency of General Relativity, eventually becoming an observable endorsement of Extended Theories of Gravity [3].

It is well known that an important potential source of gravitational radiation is the relic stochastic background of GWs [4]. The potential existence of such a relic stochastic background arises from general assumptions that mix principles of classical gravity and principles of quantum field theory [5, 6, 7]. As the zero-point quantum oscillations, which produce relic GWs, are generated by strong variations of the gravitational field in the early universe, the potential detection of relic GWs is the only way to learn about the evolution of the very early universe, up to the bounds of the Planck epoch and the initial singularity [4, 7]. The importance of this *gravity's rainbow* in cosmological scenarios has been discussed in an elegant way in [8].

The inflationary scenario for the early universe [9, 10], which is tuned in a good way with the WMAP data on the Cosmic Background Radiation (CBR) (in particular exponential inflation and spectral index ≈ 1 [11]) amplified the zero-point quantum oscillations [6, 7].

A recent paper, which has been written by the LSC [4], has shown an upper limit on the RSBGWs by using the data from a two-year science run of LIGO. Such an upper limit rules out some models of early Universe evolution, like the ones with relatively large equation-of-state parameter and the cosmic (super) string models with relatively small string tension arising from some string theory models. It results also an upper limit for the RSBGWs which is proposed by the Pre-Big-Bang Theory (see [4] for details).

Another well known upper bound on the RSBGWs arises from the Standard Inflationary Model. Such an upper bound is often updated by using the WMAP data [4, 12].

It is well known that the potential detection of such a RSBGWs is the only way to learn about the evolution of the very early universe, up to the bounds of the Planck epoch and the initial singularity. In fact, this kind of information is inaccessible to standard astrophysical observations [4, 7, 8].

In this proceeding, a formula that directly connects the average amplitude of the RSBGWs with the Inflaton field, that has been obtained in [13], will be used, together with the equation for the characteristic amplitude h_C for the RSBGWs [15], in order to translate the upper bounds on the RSBGWs from the WMAP and LSC data in lower bounds on the Inflaton field.

Our results show that the value of the Inflaton field that arises from the WMAP bound on the RSBGWs is totally in agreement with the famous *slow roll* condition on Inflation [9, 10], while the value of the Inflaton field that arises from the LSC bound on the RSBGWs could not be in agreement with this

condition.

2 The spectrum of the relic gravitational waves

Considering a stochastic background of GWs, it can be characterized by a dimensionless spectrum [4, 7, 8]. The more recent values for the spectrum that arises from the WMAP data can be found in refs. [4, 12]. In such papers it is (for a sake of simplicity, in this paper natural units are used, i.e. $8\pi G = 1$, $c = 1$ and $\hbar = 1$)

$$\Omega_{gw}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{gw}}{d \ln f} \leq 10^{-13} \quad (1)$$

where

$$\rho_c \equiv 3H_0^2 \quad (2)$$

is the (actual) critical density energy, ρ_c of the Universe, H_0 the actual value of the Hubble expansion rate and $d\rho_{gw}$ the energy density of relic GWs in the frequency range f to $f + df$. This is the upper bound on the RSBGWs that observations put on the Standard Inflationary Model.

An higher bound results from the LIGO Scientific Community data in ref. [4]:

$$\Omega_{gw} \leq 6.9 * 10^{-6}. \quad (3)$$

This bound is at 95% confidence in the frequency band $41.5 - 169.25Hz$, (see [4] for details).

In this case, the value is an upper limit for the RSBGWs which arises from the Pre-Big-Bang Theory [4, 16]. It also rules out some models of early Universe evolution, like the ones with relatively large equation of state parameter and the cosmic (super) string models with relatively small string tension arising from some string theory models (see [4] and references within).

3 Bounds from observations

Let us consider the computation in [13]. In such a paper a conformal treatment has been used to obtain:

$$\varphi = \frac{H^2}{2A_h^2} \quad (4)$$

(see Equation 42 in [13]), where φ is the Inflaton field which generates Inflation, H the value of the Hubble expansion rate at the first horizon crossing and the averaged amplitude A_h of the perturbations of the RSBGWs is defined like

$$A_h \equiv (k/2\pi)^{3/2} h \quad (5)$$

(see [13]).

We emphasize the importance of the formula (4). If the GWs interferometers will detect the RSBGWs in next years, such a formula will permit to directly compute the amount of Inflation in the early Universe. A similar computation was also performed in [14] in the framework of $f(R)$ Theories of Gravity.

The equation for the characteristic amplitude h_C is (see Equation 65 in [15])

$$h_C(f) \simeq 1.26 * 10^{-18} \left(\frac{1\text{Hz}}{f} \right) \sqrt{h_{100}^2 \Omega_{gw}(f)}, \quad (6)$$

where $h_{100} \simeq 0.71$ is the best-fit value on the Hubble constant [11]. This equation gives a value of the amplitude of the relic GWs stochastic background in function of the spectrum in the frequency range of ground based detectors [15]. Such an amplitude is also the averaged strain applied on the detector's arms by the RSBGWs [15]. Such a range is given by the interval $10Hz \leq f \leq 10KHz$ [1].

Defining the average value of $h_C(f)$ like

$$A_{hc} \equiv \frac{\int 1.26 * 10^{-18} \sqrt{h_{100}^2 \Omega_{gw}(f)} f^{-1} df}{\int df} \quad (7)$$

one can assume that it is $A_{hc} \simeq A_h$ [13].

In this way, from the fundamental eq. (4), it is also

$$\varphi \simeq \frac{H^2}{2A_{hc}^2}. \quad (8)$$

Now, by using eq. (8), we can use the bounds (1) and (3) on the RSBGWs in order to obtain bounds on the Inflaton field φ . First of all, we emphasize that a redshift correction is needed because H in eq. (8) is computed at the time of the first horizon crossing, while the value of A_{hc} from the WMAP and LSC data is computed at the present time of the cosmological Era. The redshift correction on the spectrum is well known [7]:

$$\Omega_{gw}(f) = \Omega_{gw}^0(f)(1 + z_{eq})^{-1}, \quad (9)$$

where $\Omega_{gw}^0(f)$ is the value of the spectrum at the first horizon crossing and $z_{eq} \simeq 3200$ [11] is the redshift of the Universe when the matter and radiation energy density were equal, see [7] for details.

Then, eq. (8) becomes

$$\varphi \simeq \frac{H^2}{2A_{hc}^2(1 + z_{eq})}. \quad (10)$$

By considering the WMAP bound (1), the integrals in eq. (7) has to be computed in the frequency range of ground based detectors which is the interval $10Hz \leq f \leq 10KHz$. One gets $A_{hc}^2 \simeq 10^{-51}$.

By restoring ordinary units and recalling that $H \simeq 10^{22}Hz$ at the first horizon crossing [7], at the end, from eq. (10), it is

$$\varphi \geq 10^2 \text{grams}. \quad (11)$$

This result represents a lower bound for the value of the Inflaton field that arises from the WMAP data on the RSBGWs in the case of Standard Inflation [4, 12].

Now, let us consider the LSC bound (3). Such a bound is at 95% confidence in the frequency band $41.5 - 169.25 Hz$ [4], thus, in principle, we could not extend the integrals in eq. (7) to the total interval $10Hz \leq f \leq 10KHz$. However, it is well known that for frequencies that are smaller than some hertz the spectrum which arises from the Pre-Big-Bang Theory rapidly falls, while at higher frequencies the spectrum is almost flat with a small decreasing [4, 16]. Thus, the integration of eq. (7) in the interval $10Hz \leq f \leq 10KHz$ gives a solid upper bound for A_{hc} in these models. One gets $A_{hc}^2 \simeq 10^{-44}$. In this case, by restoring ordinary units and putting the value $H \simeq 10^{22} Hz$ in eq. (10) it is

$$\varphi \geq 10^{-5} grams. \quad (12)$$

This result represents a lower bound for the value of the Inflaton field that arises from the LSC data on the RSBGWs and it has to be applied to the case of the Pre-Big-Bang Theory [4, 16].

It is well known that the requirement for inflation, which is $p = -\rho$ [9, 10], can be approximately met if one requires $\dot{\varphi} \ll V(\varphi)$, where (φ) is the potential density of the field. This leads to the famous *slow-roll approximation* (SRA), which provides a natural condition for inflation to occur [9, 10]. The constraint on $\dot{\varphi}$ is assured by requiring $\ddot{\varphi}$ to be negligible. With such a requirement, the slow-roll parameters are defined (in natural units) by [9, 10]

$$\begin{aligned} \epsilon(\varphi) &\equiv \frac{1}{2} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 \\ \eta(\varphi) &\equiv \frac{V''(\varphi)}{V(\varphi)}. \end{aligned} \quad (13)$$

Then, the SRA requirements are [9, 10]:

$$\begin{aligned} \epsilon &\ll 1 \\ |\eta| &\ll 1, \end{aligned} \quad (14)$$

that are satisfied when it is [9, 10]

$$\varphi \gg M_{Planck}, \quad (15)$$

where the Planck mass, which is $M_{Planck} \simeq 2.177 * 10^{-5} grams$ in ordinary units and $M_{Planck} = 1$ in natural units has been introduced [9, 10].

Then, one sees immediately that the value of the Inflaton field of eq. (11), that arises from the WMAP bound on the RSBGWs, is totally in agreement with the slow roll condition on Inflation. On the other hand, the value of the Inflaton field of eq. (12), that arises from the LSC bound on the RSBGWs, is of the order of the Planck mass, thus, it could not be in agreement with the slow roll condition on Inflation.

The fact that the spectrum of the RSBGWs decreases with increasing Inflaton field is not surprising. In fact, even if the amplification of zero-point quantum oscillations increases the spatial dimensions of perturbations, it is well known that the curvature of the Universe is “redshifted” by Inflation, i.e. the inflationary scenario ‘drives’ the universe to a flat geometry [9, 10].

4 Conclusion remarks

By using a formula that directly connects the average amplitude of the RSBGWs with the Inflaton field and the equation for the characteristic amplitude h_C for the RSBGWs, in this proceeding the upper bounds on the RSBGWs from the WMAP and LSC data have been translated in lower bounds on the Inflaton field.

The results show that the value of the Inflaton field that arises from the WMAP bound on the RSBGWs is totally in agreement with the famous slow roll condition on Inflation [9, 10], while the value of the Inflaton field that arises from the LSC bound on the RSBGWs could not be in agreement with such a condition.

Finally, we further emphasize the importance of the formula (4). If the GWs interferometers will detect the RSBGWs in next years, such a formula will permit to directly compute the amount of Inflation in the early Universe.

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